Signals & Systems
Homework Solutions
Problem 56.1

1(a) $m_1\ddot{d}_1 = k(d_2-d_1) - c\dot{d}_1$, $m_2\ddot{d}_2 = f + k(d_1-d_2)$

(b) States: choose $x_1 = d_1$, $x_2 = \dot{d}_1$, $x_3 = d_2$, $x_4 = \dot{d}_2$

Then $\dot{x}_1 = \dot{d}_1 = x_2$, $\dot{x}_2 = \ddot{d}_1 = \frac{k}{m_1}d_2 - \frac{k}{m_1}d_1 - \frac{c}{m_1}\dot{d}_1 = \frac{k}{m_1}x_3 - \frac{k}{m_1}x_1 - \frac{c}{m_1}x_2$

$\dot{x}_3 = \dot{d}_2 = x_4$, $\dot{x}_4 = \ddot{d}_2 = \frac{f}{m_2} + \frac{k}{m_2}d_1 - \frac{k}{m_2}d_2 = \frac{f}{m_2} + \frac{k}{m_2}x_1 - \frac{k}{m_2}x_3$

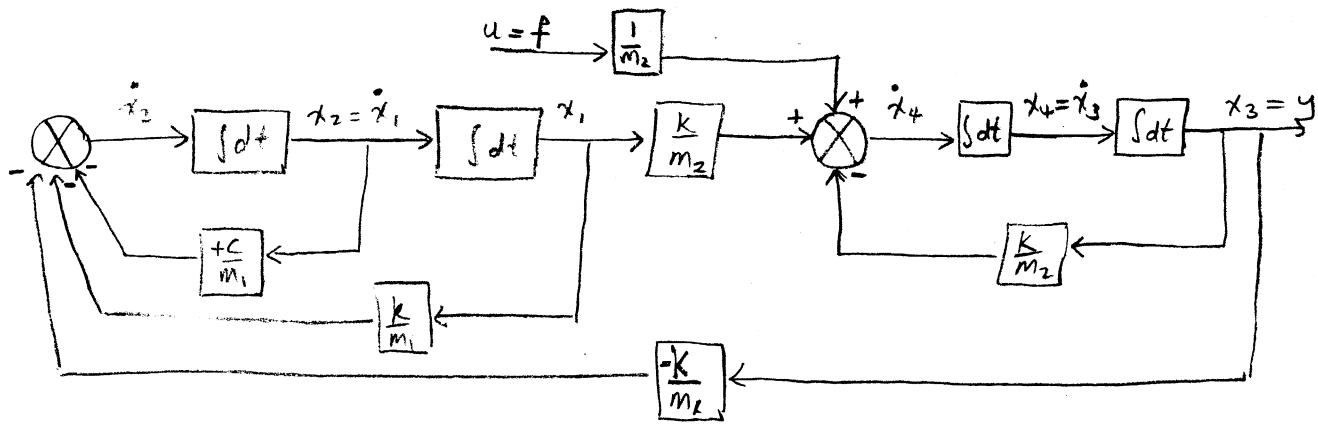
Inputs: $u = f$ Outputs: $y = d_2 = x_3$

State-space system:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{k}{m_1} & -\frac{c}{m_1} & \frac{k}{m_1} & 0\\ 0 & 0 & 0 & 1\\ \frac{k}{m_2} & 0 & -\frac{k}{m_2} & 0\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 0\\ \frac{1}{m_2}\\ 0\end{bmatrix}}_{B}u$$

$$y = \underbrace{\begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}}_{C}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \underbrace{[0]}_{D}u$$

(c)



(d) Now $\vec{y} = \begin{bmatrix}d_1\\ d_2\end{bmatrix} = \begin{bmatrix}x_1\\ x_3\end{bmatrix}$ $\Rightarrow C = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$, $D = \begin{bmatrix}0\\ 0\end{bmatrix}$

A and B unchanged from (b)

e) $\lambda_1 = 0$, $\vec{v}_1 = \begin{bmatrix}1\\ 0\\ 1\\ 0\end{bmatrix}$ $\lambda_{2,3} = -0.02 \pm 1.01j$, $v_{2,3} = \begin{bmatrix}-0.029 \pm 0.015j\\ -0.014 \mp 0.029j\\ -0.014 \mp 0.703j\\ 0.71\end{bmatrix}$ $\lambda_4 = -1.96$, $v_4 = \begin{bmatrix}-0.45\\ 0.87\\ -0.09\\ 0.18\end{bmatrix}$